Table of Integrals

A **Table of Integrals** is a compilation of integration results. A table is useful as a reference for easy as well as difficult integrals, and as a time saver, in preventing having to rework often tedious and very time-consuming derivations.

How to Use a Table of Integrals

Most integrals that you will come across, other than very elementary examples, will not look exactly the same as a result in a table. It may be necessary to try to rearrange the integrand into a specific form, simplify it by algebraic manipulations, trigonometric identities, other identities or a combination of these techniques. The point of rearranging the integrand is so that it takes the *general form* of an integral in a table.

Examples of How to Use This Table of Integrals

Example 1: Use the Table of Integrals to calculate $I_1 = \int \frac{\sqrt{8x^2-3}}{x^2}\,dx.$

The integral in the Table that looks most like the integral above, is IA202, found in Integrands containing $\sqrt{x^2-a^2}$, in the Algebraic Integrals section:

$$\int \frac{\sqrt{x^2-a^2}}{x^2}\,dx = -\frac{1}{x}\sqrt{x^2-a^2} + \ln\left|x+\sqrt{x^2-a^2}\right| + C.$$

The main parts of the integrand of I_1 are similar to those in IA202, so use the substitution $u=\sqrt{8}x$, then $x=\frac{u}{\sqrt{8}}$ and $dx=\frac{1}{\sqrt{8}}du.$

Substituting:

$$I_1 = \int \frac{\sqrt{8x^2-3}}{x^2}\,dx = \int \frac{\sqrt{\left(\sqrt{8}x\right)^2-3}}{x^2}\,dx = \int \frac{\sqrt{u^2-3}}{\left(\frac{u}{\sqrt{8}}\right)^2}\cdot\frac{1}{\sqrt{8}}\,du = \int \frac{\sqrt{u^2-3}}{\frac{u^2}{\left(\sqrt{8}\right)^2}}\cdot\frac{1}{\sqrt{8}}\,du = \sqrt{8}\int \frac{\sqrt{u^2-3}}{u^2}\,du,$$

which now has the exact same form as IA202 from the Table, with the exception of the $\sqrt{8}$ multiplier. Now apply IA202, with $a^2=3$:

$$I_1 = \int \frac{\sqrt{8x^2-3}}{x^2}\,dx = \sqrt{8}\int \frac{\sqrt{u^2-3}}{u^2}\,du = \sqrt{8}\left[-\frac{1}{u}\sqrt{u^2-3} + \ln\left|u+\sqrt{u^2-3}\right|\right] + C.$$

Rewriting in terms of *x*:

$$I_1 = \sqrt{8}\left[-\frac{1}{\sqrt{8}x}\sqrt{\left(\sqrt{8}x\right)^2-3} + \ln\left|\sqrt{8}x+\sqrt{\left(\sqrt{8}x\right)^2-3}\right|\right] + C = \sqrt{8}\left[-\frac{1}{\sqrt{8}x}\sqrt{8x^2-3} + \ln\left|\sqrt{8}x+\sqrt{8x^2-3}\right|\right] + C,$$

and finally,

$$I_1 = -\frac{1}{x}\sqrt{8x^2-3} + \sqrt{8}\ln\left|\sqrt{8}x+\sqrt{8x^2-3}\right| + C.$$

Example 2: Use the Table of Integrals to calculate $I_2 = \int \frac{\sin(x)\cos(x)}{\sqrt{\sqrt{2}+\sin(x)}}\,dx.$

Though your first inclination might be to try to use IT143, from the Trigonometric Table of Integrals, $\int \frac{\cos(ax)}{\left[m+n\sin(x)\right]^r}\,dx,$ with $a=1,\ m=\sqrt{2},\ n=1$ and $r=\frac{1}{2}$, IT143 is not close enough to I_2 since it lacks the $\sin(x)$ factor. But if you think *first* in terms of substitution, you can let $u=\sin(x),\ du=\cos(x)dx,$ then IA38 $\int \frac{x}{\sqrt{a+bx}}\,dx = \frac{2(bx-2a)}{3b^2}\sqrt{a+bx}+C,$ from the *Algebraic* Table of Integrals is an exact fit. Rewriting IA38 in terms of *u*, putting $a=\sqrt{2},\ b=1$ and applying the substitution for $\sin(x)$:

$$\int \frac{\sin(x)\cos(x)}{\sqrt{\sqrt{2}+\sin(x)}}\,dx = \int \frac{u}{\sqrt{\sqrt{2}+u}}\,du \text{ and } I_2 = \frac{2\left[(1)u-2\sqrt{2}\right]}{3(1)^2}\sqrt{\sqrt{2}+(1)u}+C = \frac{2\left[u-2\sqrt{2}\right]}{3}\sqrt{\sqrt{2}+u}+C.$$

Converting back to trigonometric form:

$$I_2 = \frac{2\left[\sin(x)-2\sqrt{2}\right]}{3}\sqrt{\sqrt{2}+\sin(x)}+C = \frac{2}{3}\left[\sin(x)-2\sqrt{2}\right]\sqrt{\sqrt{2}+\sin(x)}+C.$$